LJ ①

S6 MATHS PAPER TWO MARKING GUIDE – MOCK 2 2017

1. $F(x) = \begin{cases} 0 & ; x < 1 \\ \frac{1}{4}(x-1)^2 & ; 1 \le x \le 3 \\ 1 & ; x > 3 \end{cases}$

(i) $P(1.5 \le x \le 2) = F(2) - F(1.5)$

$= \frac{1}{4}(2-1)^2 - \left[\frac{1}{4}(1.5-1)^2\right]$. M1 M1

$= \frac{1}{4} - \frac{1}{16}$

$= \frac{3}{16}$ A1 or 0.1875

(ii) For $x<1$, $f(x) = 0$.

For $1 \le x \le 3$, $f(x) = \frac{1}{4}\cdot 2(x-1) = \frac{1}{2}(x-1)$ M1

For $x \ge 3$, $f(x) = 0$.

$\therefore f(x) = \begin{cases} \frac{1}{2}(x-1) & ; 1 \le x \le 3 \\ 0 & , \text{elsewhere} \end{cases}$ A1

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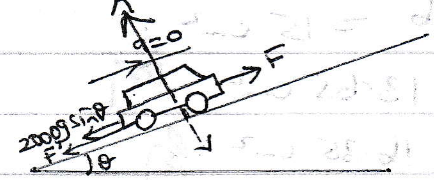
2. $m = 2000\,kg$; $P = 45 \times 10^3\,W$; $\theta = \sin^{-1}\frac{1}{70}$; $F' = 300\,N$

(i)



F = Driving force.

$F - (2000g\sin\theta + F') = 2000 \times 0$.

$F = 2000 \times 9.8 \times \frac{1}{70} + 300$ M1

$F = 580\,N$ A1

(ii) When $V = 50\,ms^{-1}$, $P = FV$

$F = \frac{45 \times 10^3}{50} = 900\,N$. B1

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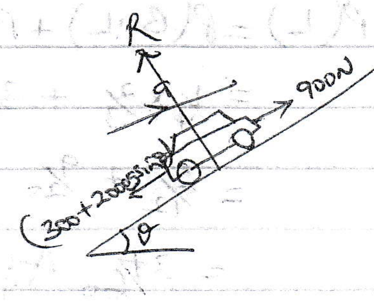


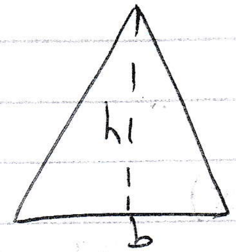
$900 - (300 + 2000 \times 9.8 \times \frac{1}{70}) = 2000a$. M1

$a = 0.16\,ms^{-2}$ A1

OR $900 - 580 = 2000a$ ✓

$a = 0.16\,ms^{-2}$ ✓

3.



(i) $b = 5\text{cm}$; $h = 6\text{cm}$.

$\left|\frac{\Delta b}{b}\right| = 0.04$

$|\Delta b| = 0.04 \times 5 = 0.2\text{ cm}$ B1

(ii) $\left|\frac{\Delta h}{h}\right| = 0.05$.

$|\Delta h| = 0.05 \times 6$

$= 0.3\text{ cm}$ B1

(ii) $A = \frac{1}{2}bh$.

$A_{max} = \frac{1}{2} \times (5+0.2)(6+0.3) = 16.38\text{ cm}^2$ B1

$A_{min} = \frac{1}{2}(5-0.2)(6-0.3) = 13.68\text{ cm}^2$. B1

Range $= (13.68, 16.38)$ B1

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ALT:

$A = \frac{1}{2}bh$.

$|\Delta A| = \frac{1}{2}[|\Delta b|h + |\Delta h|b] = \frac{1}{2}[0.2 \times 6 + 0.3 \times 5] = 1.35$. $\cancel{\frac{1}{2}[0.2+0.3]}$

$\cancel{= 0.25}$.

Working value $= \frac{1}{2} \times 5 \times 6 = 15\text{ cm}^2$.

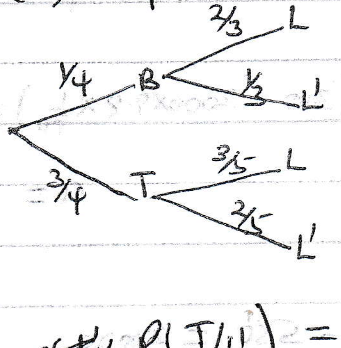
Min. value $= 15 - 1.35 = 13.65\text{ cm}^2$. B1

max. value $= 15 + 1.35 = 16.35\text{ cm}^2$. B1

Range $= (13.65, 16.35)$. B1

4. Let B = event she uses a bus; T = event she uses a taxi ; L = event she is late

$P(B) = \frac{1}{4}$



(i) $P(L) = P(B \cap L) + P(T \cap L)$

$= \frac{1}{4} \times \frac{2}{3} + \frac{3}{4} \times \frac{3}{5}$ M1B1

$= \frac{2}{12} + \frac{9}{20}$

$= \frac{37}{60}$ A1

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(ii) $\cancel{P(L')}$ $P(T/L') = \frac{P(T \cap L')}{P(L')}$

5\. (i)

4	7	10
2800	a	5800

B1

$$\frac{a-2800}{7-3} = \frac{5800-2800}{10-4} \quad M1$$

$$a = 2800 + \frac{3000 \times 4}{6}$$

$$= 4{,}800k \quad A1$$

(ii)

4	d	10
2800	5300	5800

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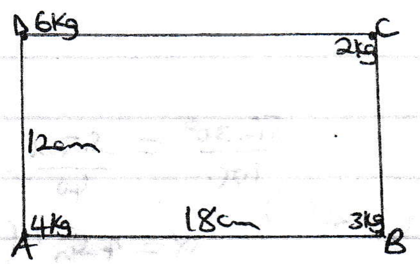
$$\frac{d-4}{5300-2800} = \frac{10-4}{5800-2800} \quad M1$$

$$d = 4 + \frac{6 \times 2500}{3000}$$

$$= 9 \text{ Km} \quad A1$$

6\.



Total weight $= 4g + 3g + 2g + 6g$

$= 15g$ N . B1

Taking moments about AD;

$3g \times 18 + 2g \times 18 = 15g\bar{x}$. M1

$\bar{x} = 6$ cm to the right of AD . A1

05

Taking moments about AB;

$6g \times 12 + 2g \times 12 = 15g\bar{y}$. M1

$\bar{y} = 6.4$ cm above AB A1

ALT 1:

$$4g\begin{pmatrix}0\\0\end{pmatrix} + 3g\begin{pmatrix}18\\0\end{pmatrix} + 2g\begin{pmatrix}18\\12\end{pmatrix} + 6g\begin{pmatrix}0\\12\end{pmatrix} = 15g\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} \quad M1 \quad M1$$

$\bar{x} = 6$ A1; $y = 6.4$ A1

$\therefore$ C.o.g is 6cm to right of AD and 6.4 cm above AB. B1

7.

Marks	f	F	x	fx
30–	8	8	35	280
40–	5	13	45	225
50–	12	25	55	660
60–	9	34	65	585
70–	6	40	75	450
80–	0			$\Sigma fx = 2200$

B1

(i) mode $= 50 + \frac{(12-5)}{(12-5)+(12-9)} \times 10$ M1

$= 50 + \frac{7}{10} \times 10$

$= 57.$ A1

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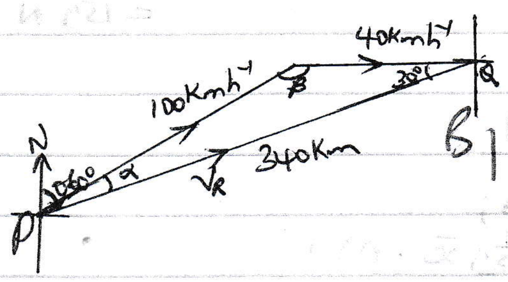
(ii) $\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{2200}{40}$ M1 $= 55$ A1

8.



B1 (sketch)

$\frac{\sin 30^\circ}{100} = \frac{\sin \alpha}{40}$

$\alpha = \sin^{-1}\left(\frac{40 \sin 30^\circ}{100}\right)$

$= 11.54^\circ.$ B1

$\beta = 180^\circ - (11.54^\circ + 30^\circ) = 138.46^\circ$

$\frac{V_R}{\sin 138.46^\circ} = \frac{100}{\sin 30^\circ}$

$V_R = 132.6286 \text{ Kmh}^{-1}$ B1

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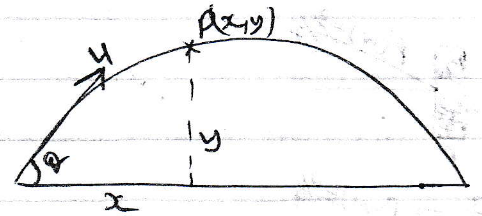
$t = \frac{340}{132.6286}$ M1

$= 2.5635 \text{ hrs}$ A1

Accept alternative.

9. (a)



Horizontal distance travelled after a time, t

$x = u\cos\theta \cdot t$. M1

$t = \dfrac{x}{u\cos\theta}$ ——— (i).

vertical distance, $y = u\sin\theta \cdot t - \frac{1}{2}gt^2$ ——— (ii). M1

From (i) and (ii);

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2. \quad M1$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad A1$$

(b) $y = 12\,m$; $x = 32\,m$.

using $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$$12 = 32\tan\theta - \frac{9.8(32)^2}{2u^2\cos^2\theta} \text{ ——— (i) } M1$$

But in a horizontal direction, $v = 0$.

From $v^2 = u^2\sin^2\theta - 2gh$.

$$0 = u^2\sin^2\theta - 2 \times 9.8 \times 12 \quad M1$$

$$u^2 = \frac{235.2}{\sin^2\theta} \text{ ——— (ii) } A1$$

Substitute (ii) in (i);

$$12 = 32\tan\theta - \frac{9.8 \times 1024}{2 \times \frac{235.2}{\sin^2\theta}\cos^2\theta} \quad M1$$

$$12 = 32\tan\theta - \frac{64}{3}\tan^2\theta.$$

$$64\tan^2\theta - 96\tan\theta + 36 = 0 \quad A1$$

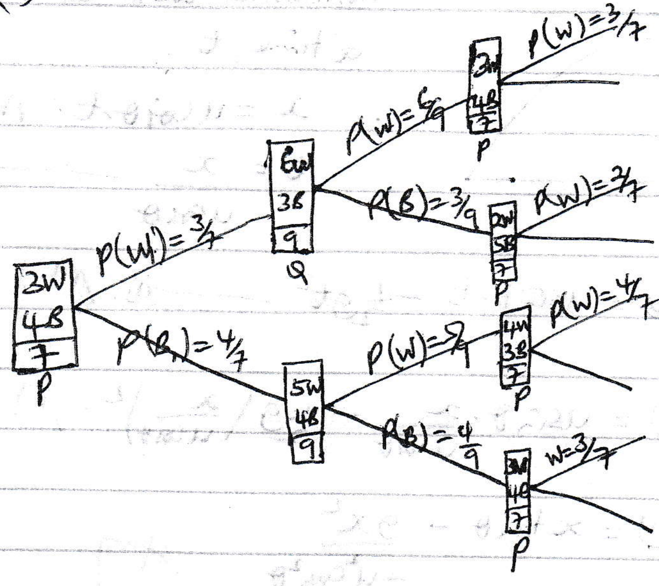
$$\tan\theta = \frac{96 \pm \sqrt{96^2 - 4 \times 64 \times 36}}{2 \times 64} \quad M1$$

$$= 0.75 \text{ or } 0.75.$$

$$\theta = \tan^{-1} 0.75 = 36.87^\circ \quad A1$$

M1

10. (a)

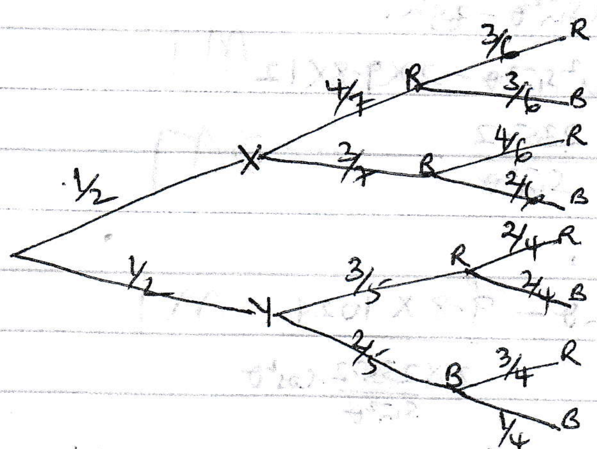


$$P(W) = \tfrac{3}{7}\times\tfrac{6}{9}\times\tfrac{3}{7} + \tfrac{3}{7}\times\tfrac{3}{9}\times\tfrac{2}{7} + \tfrac{4}{7}\times\tfrac{5}{9}\times\tfrac{4}{7} + \tfrac{4}{7}\times\tfrac{4}{9}\times\tfrac{3}{7}$$ B1 B1 B1 M1 (add'n)

$$= \frac{54}{441} + \frac{18}{441} + \frac{80}{441} + \frac{48}{441}$$

$$= \frac{200}{441}$$ A1 (or 0.4535)

(b)



(i) $$P(\text{different colours}) = \tfrac{1}{2}\times\tfrac{4}{7}\times\tfrac{3}{6} + \tfrac{1}{2}\times\tfrac{3}{7}\times\tfrac{4}{6} + \tfrac{1}{2}\times\tfrac{3}{5}\times\tfrac{2}{4} + \tfrac{1}{2}\times\tfrac{2}{5}\times\tfrac{3}{4}$$ B1 B1 M1

$$= \frac{12}{84} + \frac{12}{84} + \frac{6}{40} + \frac{6}{40}$$

$$= \frac{41}{70}$$ A1 (or 0.5857)

(ii) $P(Y/S) = \frac{P(Y\cap S)}{P(S)}$; S = Same colour.

$P(S) = 1 - 41/70 = 29/70$ B_1 ($P(S)$)

$P(Y/S) = \frac{P(Y\cap S)}{P(S)} = \frac{\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4} + \frac{1}{2}\times\frac{3}{5}\times\frac{1}{4}}{29/70}$ M_1

03

$= 14/29$ A_1

12

11. (a) $y = \tan x - 4x$

x	1	1.1	1.2	1.3	1.4	1.5
y	-2.44	-2.435	-2.2	-1.6	0.2	8.1

B_2 (atleast 3 points)

(b) $f(x) = \tan x - 4x$.

$f'(x) = \sec^2 x - 4 \overset{M_1}{=} \tan^2 x - 3$.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{[\tan x_n - 4x_n]}{\tan^2 x_n - 3}$ M_1

$= \frac{x_n \tan^2 x_n - 3x_n - \tan x_n + 4x_n}{\tan^2 x_n - 3}$

$= \frac{x_n \tan^2 x_n - \tan x_n + x_n}{\tan^2 x_n - 3}$ A_1

$x_0 = 1.39$

$x_1 = \frac{1.39\tan^2 1.39 - \tan 1.29 + 1.39}{\tan^2 1.39 - 3} = 1.3933$ B_1

$x_2 = \frac{1.3933\tan^2 1.3933 - \tan 1.3933 + 1.3933}{\tan^2 1.2933 - 3} = 1.3932$ B_1

12.

Marks	f	F	x	x^2	fx^2	c	f·d	
<5	6	6	2.5	6.25 ~~15~~	37.5	5	1.2	[illegible]
<10	11	17	7.5	56.25	618.75	5	2.2	82
<15	17	34	12.5	156.25	2,656.25	5	3.4	212.5
<25	28	62	20	400	11,200 B1	10	2.8	560
<30	20	82	27.5	756.25	15,125	5	4	550 B
<35	15	97	32.5	1,056.25	15,843.75	5	3	487.5
<40	3	100	37.5	1,406.25	4,218.75	5	0.6	112.5
					$\Sigma fx^2 =$ 49,700		B1 all	$\Sigma fx =$ 2,020

(a) (i) $Q_3 = 25 + \left(\frac{\frac{3}{4}\times 100 - 62}{20}\right)\times 5$ M1

$= 25 + \frac{13\times 5}{20}$

$= 28.125$ A1

(ii) $\sigma^2 = \frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2$

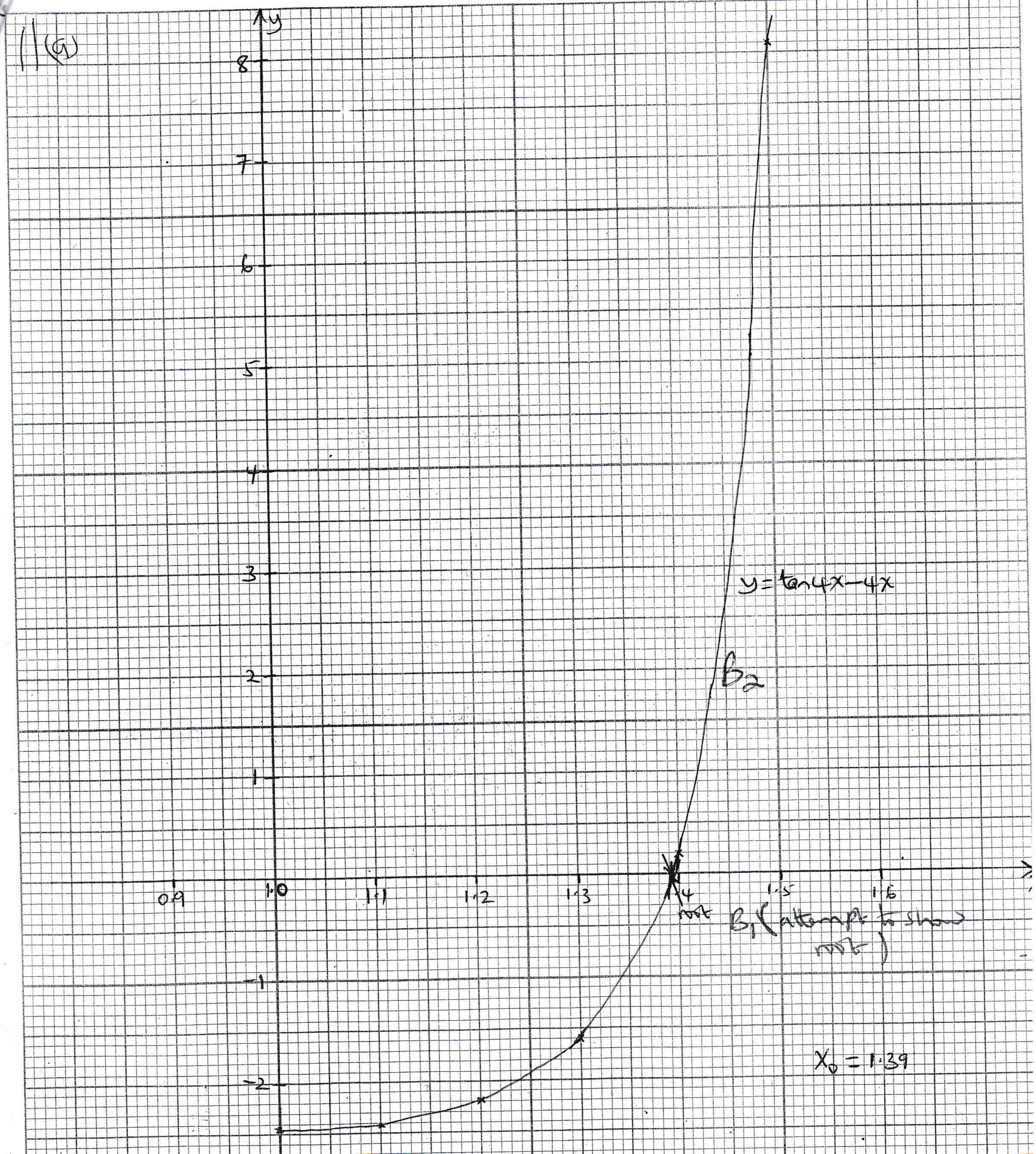
$= \frac{49,700}{100} - \left(\frac{2,020}{100}\right)^2$ M1

$= 88.96$ A1

12

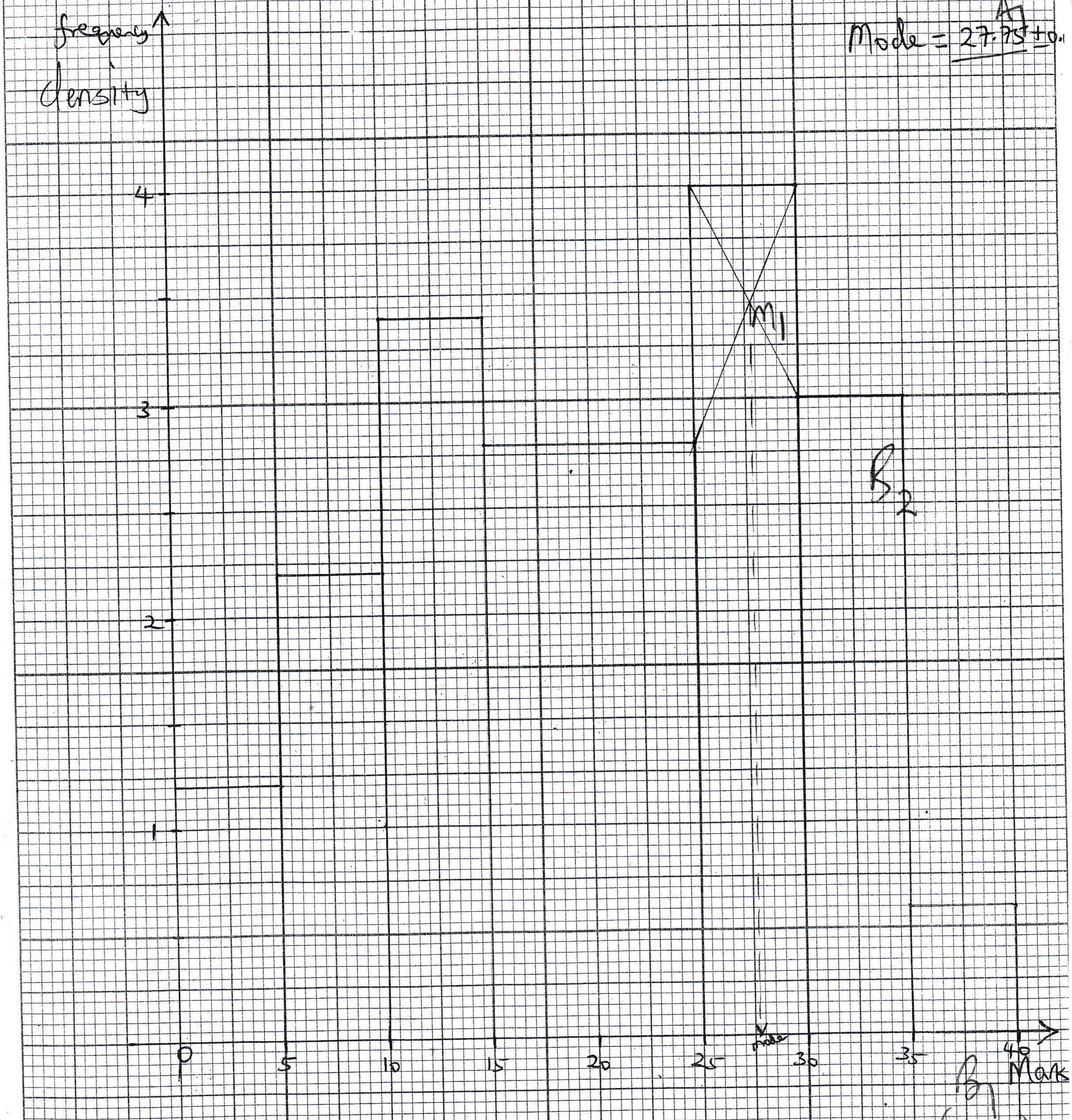
KING'S COLLEGE BUDO

Name:..Class...........................Index Number...........................



(12) (b)

Mode = $\underline{27.75 \pm 0.1}$ A1



13. (a) $a = -w^2 x$.

$$\frac{d^2x}{dt^2} = -w^2 x$$

$$\frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = -w^2 x.$$

$$v\frac{dv}{dx} = -w^2 x.$$

$$\int v\,dv = \int -w^2 x\,dx. \quad M1$$

$$\frac{v^2}{2} = -\frac{w^2x^2}{2} + c_1 \quad A1$$

At $x = A$, $V = 0 \Rightarrow c_1 = \frac{w^2A^2}{2}$ B1

$$\frac{v^2}{2} = \frac{w^2}{2}(A^2 - x^2).$$

$$v^2 = w^2(A^2 - x^2).$$

$$V = w\sqrt{A^2 - x^2}.$$

$$\frac{dx}{dt} = w\sqrt{A^2 - x^2}. \quad M1$$

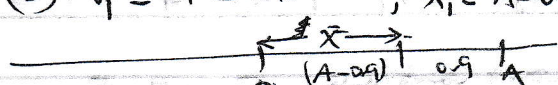
$$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int w\,dt$$

$$\sin^{-1}\left(\frac{x}{A}\right) = wt + \varepsilon \quad A1$$

$$\frac{x}{A} = \sin(wt + \varepsilon).$$

$$x = A\sin(wt + \varepsilon) \quad A1$$

(b) $V_1 = 7.5\,ms^{-1}$; $X_1 = A - 0.9$



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$$v^2 = w^2(A^2 - x^2).$$

$$7.5^2 = w^2[A^2 - (A - 0.9)^2] \quad m1 \ (i).$$

For $V_2 = 4\,ms^{-1}$; $x_2 = A - 0.2$

$$4^2 = w^2[A^2 - (A - 0.2)^2] \quad m1 \ (ii).$$

(i) ÷ (ii) ; $\frac{7.5^2}{4^2} = \frac{A^2 - (A-0.9)^2}{A^2 - (A-0.2)^2}$ M1 (div)

$$A = \frac{64\times0.81 - 225\times0.04}{64\times1.8 - 225\times0.4}$$

~~A = 0.4.~~

$A = 1.7\,m$ A1

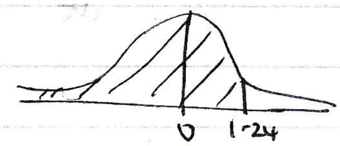
Length of path

14. (a) $n = 400$ books ; $\mu = 50$ kg, $\sigma = 2g$.

Let $X \sim N(50, 4)$

$P(X < 52.48) = P\left(Z < \frac{52.48 - 50}{2}\right)$ M1 (method of standardizing)

$= P(Z < 1.24)$



$= 0.5 + \underline{0.3925}$ B1

$= 0.8925$ A1

No of books $= 400 \times 0.8925$ M1

$= \underline{357}$. A1

(b) $n = 100$; $\bar{X} = 82$; $\sum X^2 = 686{,}800$.

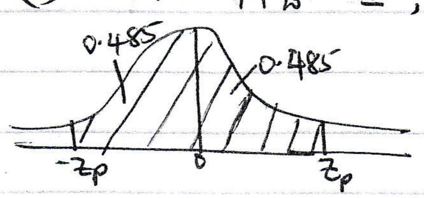
(i) $\sigma_x = \sqrt{\frac{n}{n-1}\left[\frac{\sum X^2}{n} - (\bar{X})^2\right]}$

$= \sqrt{\frac{100}{100-1}\left[\frac{686{,}800}{100} - 82^2\right]}$ M1

$= \sqrt{145.454545}$

$= \underline{12.0605}$ A1

(ii) For 97% CI,



$z_p = 2.17$ B1 (or $z_p = 2.1705$)

Lower confidence limit $= \bar{X} - z_p \frac{\sigma_x}{\sqrt{n}}$.

$= 82 - \frac{2.17 \times 12.0605}{\sqrt{100}}$ M1

$= 79.3829$ A1 (both limit

Upper confidence limit $= 82 + \frac{2.17 \times 12.0605}{100}$ M1

$= 84.6171$ A1

15. (a) $h = \frac{3-1}{6-1} \overset{B_1}{=} 0.4$; $y = \sin^2 x$

x	y_0, y_5	$y_1, y_2, \ldots$
1	0.7081	
1.4		0.9711
1.8		0.9484
2.2		0.6537
2.6		0.2657
3	0.0199	
B_1 (all)	0.7280	2.8389

B_2 (all ordinates correct)

$$\int_1^3 \sin^2 x \, dx \simeq \frac{1}{2} \times 0.4 \left[0.7280 + 2 \times 2.8389\right] \quad M_1$$

$$= 1.281 \quad A_1$$

(b) $$\int_1^3 \sin^2 x \, dx = \int_1^3 \frac{1}{2}(1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_1^3 \quad M_1$$

$$= \frac{1}{2}\left(\left[3 - \tfrac{1}{2}\sin 6\right] - \left[1 - \tfrac{1}{2}\sin 2\right]\right) \quad M_1 \text{ (subst'n)}$$

$$= 1.297 \quad A_1$$

$$= \cancel{1.594}$$

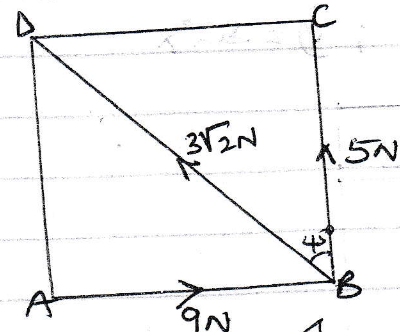
(c) %ge error $= \frac{(1.297 - 1.281)}{1.297} \times 100 \quad M_1$

$= 1.2336\% \quad A_1$

(Accept ≥ 2dp)

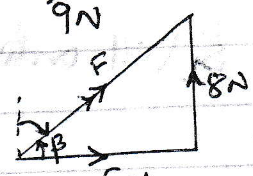
- Increasing the number of strips / ordinates B_1
- Reducing the width of each strip.

16. (a)



$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 3\sqrt{2}\cos 135° \\ 3\sqrt{2}\sin 135° \end{pmatrix}$$ M1 M1

$$= \begin{pmatrix} 6 \\ 8 \end{pmatrix}.$$



Resultant, $F = \sqrt{6^2 + 8^2}$

$= 10N$. B1

(05)

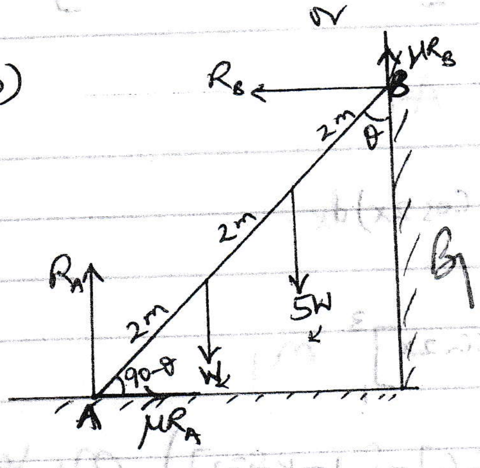
$\tan\beta = 8/6$. M1

$\beta = \tan^{-1}(8/6) = 53.13°$.

Direction is E 53.13° N A1

or N 36.87° E

(b)



Let R_A and R_B be the normal reaction at A and B respectively.

At limiting equilibrium;

$\uparrow$; $R_A + \mu R_B = W + 5W$. M1

$R_A + \mu R_B = 6W$ ——— (i).

$\rightarrow$ $R_B = \mu R_A$ ——— (ii).

From (i) and (ii)

$R_A + \mu^2 R_A = 6W$.

$R_A = \dfrac{6W}{1+\mu^2}$. A1

$R_B = \dfrac{6\mu W}{1+\mu^2}$ A1

(07)

Taking moments about the ground at A; M1

$W \cdot 2\cos(90-\theta) + 5W \cdot 4\cos(90-\theta) = R_B \cdot 6\sin(90-\theta) + \mu R_B \cdot 6\cos(90-$

$2W\sin\theta + 20W\sin\theta = \dfrac{36W\mu}{1+\mu^2}\cos\theta + \dfrac{36\mu^2 W}{1+\mu^2}\sin\theta$ M1

~~$24\tan\theta =$~~

$\dfrac{22(1+\mu^2) - 36\mu^2}{1+\mu^2}\sin\theta = \dfrac{36\mu}{1+\mu^2}\cos\theta$.

$\tan\theta = 18\mu$ A1